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A Constrained Search Algorithm for Selection of Optimal Generators in Convolutional Encoder

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Abstract— **In this paper, we propose an efficient constrained search method for the selection of optimal combination of generator polynomials in convolutional encoders, for various code rates and constraint lengths. The algorithm is very helpful for finding optimal generators in higher constraint length coders, as it reduces the search space. This plays a vital role in the error performance of the coder.**

Keywords— **Channel coding, Convolutional codes, Generator polynomials, Optimization, Distance Spectrum.**

I. Introduction

As convolutional codes are well-known for their error-correcting properties, they are widely used in digital communication systems. The performance of the coder varies with the generator polynomials for various code-rates. Hence, to have a convolutional encoder that has good error performance, it is very important to choose good generator polynomials.

II. Convolutional Encoder

A Convolutional encoder encodes a certain input depending upon the state in the memory registers. The output depends upon the input and the previous state. The state transitions and the outputs are given by the trellis diagram. Depending upon the path metric and branch metric values, the free distance of the coder is computed. It is the distance of the all-zero sequence from its nearest neighbouring codeword. It is also the maximum value of the column distance in the trellis.

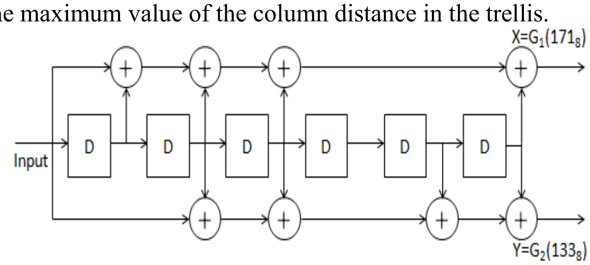

Fig. 1 Standard IEEE 802.11 Convolutional Encoder

Convolutional codes are found to be good when the free distance is large. Hence, the previous works aim at obtaining generator polynomials with maximum free distance. However, there occur multiple generator combinations that yield the maximum free distance. Hence, restricting the selection of polynomials to the maximum free distance alone will not be a good choice.

III. Optimal Generator Polynomials

A. *Parameters Affecting Performance*

Other than the free distance of the coder, the number of information bit errors and the number of error events also determine the coder's performance in communication system. Hence, we need generator polynomials with maximum free distance, minimum information error weight and minimum error event. These three components together comprise the distance spectrum of the convolutional coder.

B. *Distance Spectrum*

Though there are various generator combinations yielding the maximum free distance, they differ in their distance spectrum values. Considering the first component in the spectrum (the so called ODS1 criteria), most generators yield optimal value. However, there may be a few cases where more than two generator combinations share the same distance spectrum value in the first component of the spectrum. Hence we go for higher components to find out the generators yielding minimal value.

IV. Algorithm for Optimization

To select optimal generators for a certain constraint length rate $1/v$ convolutional encoder, a constrained algorithm that reduces the search space of the generators is proposed, and is shown below:

- Obtain all possible bit combinations along the constraint length, excluding extreme bit-positions. This gives the individual mid-generators.
- Obtain mid-generator combinations for the required code-rate
- Eliminate redundant combinations on bit-reversal
- Append '1' to both sides of individual mid-generators, to get the original generators
- Eliminate invalid & catastrophic generators by passing through the trellis
- Identify maximum d_{free} generators
- Extract generators with minimum information error weight
- Further extract the set of generators with minimum error event in the first component of the distance spectrum (ODS1).
- If more than one set of generators result, then, go for the next higher component in the distance spectrum. Repeat this step until a single set of optimal generators is obtained.

V. Detailed Methodology

For a constraint length K, it is possible to have $2^K - 1$ generators. However, to reduce the search, the algorithm tends to minimize the initial generators to be considered. Instead of performing an exhaustive search, it will be sufficient to have generators whose first and last terms are always equal to '1'.

Hence, we consider only $K-2$ terms for the generators. (Let us use the term mid-generator, to denote this.) By this, much of the search is reduced.

For a generator $G_j(D) = \sum_{k=0}^{m} g_k D^k$, we have $g_0 = g_m = 1$, where $j = g_1 + 2g_2 + \cdots + 2^{m-2} g_{m-1}$

Here, we have to consider only the values that $\boldsymbol{j}$ take, to obtain the possible mid-generators. This reduces the sample space of generators by 4 times.

From this, we obtain all possible mid-generator combinations and then eliminate redundancies. i.e., even if the bits of the mid-generators are reversed, they should not yield the same mid-generators obtained prior to them. This is true, as the octal representation of the generator polynomials can be written by reading the convolutional encoder from either ends. To this result, we now append a '1' on both sides of all the mid-generators in the combinations. This gives us the actual generator combinations.

Now, we eliminate the invalid and catastrophic generator polynomial combinations, by checking their trellises. This step is necessary, as we do not want generator combinations that yield all-zero outputs. This gives the sample space of the generators to be checked for optimal distance spectrum.

The procedure that follows is simple. The generators are checked for maximum free distance. For these MFD generators, we check the ODS1 criteria. This means that that the number of bit errors and the error events are checked for a minimum. But, obtaining the best generator at ODS1 is difficult. Hence, if more than one generator satisfies the ODS1 criteria, we go for higher components of the distance spectrum for the few final generators alone.

VI. Optimization Results

The algorithm for optimization of the set of generators is run for rates ½, ⅓ and ¼, for various constraint lengths of the coders. Restricting to constraints, the possible set of generators are checked for redundancy, and the valid non-catastrophic generators are optimized for maximum free distance, minimum information error weight and minimum error event. The software used for this is MATLAB R2008a.

A. Rate ½

Optimization is done for rate ½ convolutional encoders, where only two generators constitute a set. For a constraint length of K, we have 2^K ways of writing a generator. So, for ½ rate coder, we have $2^{K-1}(2^K - 1)$ ways for its set of generators, in the exhaustive search method.

In this constrained algorithm, one generator has 2^{K-2} ways, and hence, a rate ½ coder has only $2^{K-3}(2^{K-2} - 1)$ ways for its set of generators, which is significantly very much lesser than the exhaustive search case. The table I indicates the number of ways by which the search is reduced at each stage. It also shows the list of all optimal sets of generators for constraint lengths ranging from 3 to 10. Most of them satisfy the optimization criteria at the first component of the distance spectrum itself. Generators for K=8, 10 reach optimization at the second component, and K=5 at the third component.

B. Rate ⅓

Optimization is done for rate ⅓ convolutional encoders, where three generators constitute a set. The table II shows how much of a vast difference exists between the exhaustive search and the proposed constrained search method, for constraint lengths ranging from 3 to 8. The optimal generators for the rate ⅓ coder are also presented. The generators for constraint lengths K=7 and 8 satisfy the optimum distance criteria at the second component of their distance spectrum.

C. Rate ¼

The constrained search algorithm for optimization is run for rate ¼ encoder with constraint lengths ranging from 3 to 7, and the results are shown in the table III. The last column shows the selected generators, which are found to be the optimal ones, of all its possible sets of generators. We can also observe that, for constraint lengths 6 and 7, optimal generators are obtained at the third component of the distance spectrum.

TABLE I
SEARCH RESULTS FOR RATE ½ CODER

K	No. of generator sets		Non-redundant	Valid	Instances		Max D_{free}	Min info error weight-event	ODS	G1-G2
	Exhaustive	Constrained			MFD	Min error weight				
3	28	1	1	1	1	1	5	1-1	1	5-7
4	120	6	4	3	2	1	6	2-1	1	13-17
5	496	28	18	13	3	2	7	4-2,12-3,20-4	3	23-35
6	2016	120	66	44	17	1	8	2-1	1	53-75
7	8128	496	268	181	2	1	10	36-11	1	117-155
8	32640	2016	1036	688	127	3	10	2-1,22-6	2	237-345
9	130816	8128	4152	2773	18	1	12	33-11	1	435-657
10	523776	32640	16440	10944	1028	10	12	2-1,21-7	2	1131-1537

TABLE II
SEARCH RESULTS FOR RATE ⅓ CODER

K	No. of generator sets		Non-redundant	Valid	Instances		Max D_{free}	Min info error weight-event	ODS	G1-G2-G3
	Exhaustive	Constrained			MFD	Min error weight				
3	112	2	2	2	1	1	8	3-2	1	5-7-7
4	800	16	10	8	2	1	10	6-3	1	13-15-17
5	5952	112	68	56	1	1	12	12-15	1	25-33-37
6	45696	800	420	342	14	1	13	1-1	1	47-53-75
7	357632	5952	3080	2584	17	3	15	7-3,8-3	2	117-127-155
8	2828800	45696	23016	19462	1425	3	16	1-1,0-0,24-8	3	225-331-367

TABLE III
SEARCH RESULTS FOR RATE ¼ CODER

K	No. of generator sets		Non-redundant	Valid	Instances		Max D_{free}	Min info error weight-event	ODS	G1-G2-G3-G4
	Exhaustive	Constrained			MFD	Min error weight				
3	322	3	3	3	2	1	10	1-1	1	5-5-7-7
4	3860	31	19	16	1	1	13	4-2	1	13-13-15-17
5	52328	322	188	166	1	1	16	8-4	1	25-27-33-37
6	766416	3860	1986	1779	30	6	18	5-3,0-0,9-3	3	45-53-73-77
7	11716512	52328	26580	24391	537	10	20	3-2,0-0,17-6	3	117-127-155-171

D. Error Performance Plots

The plot of E_b/N_o versus bit error rate is plotted using hard-decision decoding function for code-rates ½, ⅓ and ¼ as an upper bound on bit error rates, and the performance is compared between maximum free distance generators and the optimal distance spectrum generators. It is found that the ODS generators have better error performance than the maximum free distance generators. The plots are shown below:

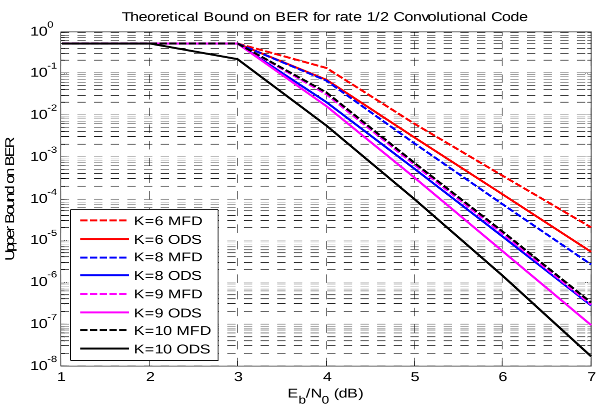

Fig. 2 Plot of E_b/N_o versus BER for a rate ½ convolutional encoder

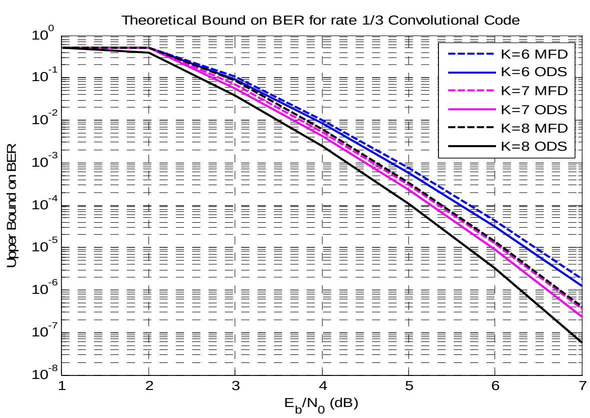

Fig. 3 Plot of E_b/N_o versus BER for rate ⅓ convolutional encoder

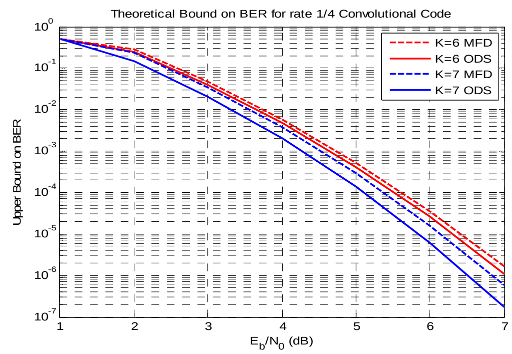

Fig. 4 Plot of E_b/N_o versus BER for rate ¼ convolutional encoder

VII. CONCLUSIONS

The algorithm hence proposed in this paper uses a constrained search method, to effectively optimize the generator polynomials for convolutional codes. This method is especially helpful for higher constraint length coders, where the number of states and the trellis search are all huge. It outperforms the other methods of obtaining optimal generators, as it does not perform an exhaustive search, and keeps the search to a minimum, limited by constraints.

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